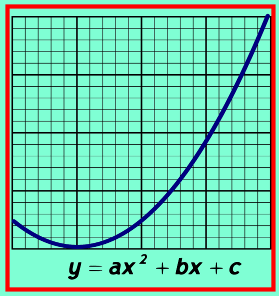
Math 125

Fall 2021

Lecture 32



Class QZ 26:

Solve by Cramer's rule for Y-only

$$\begin{cases} 4x - 3y = 14 \\ 3x - y = 3 \end{cases}$$

$$D = \begin{vmatrix} 4 & -3 \\ 3 & -1 \end{vmatrix} = 4(-1) - 3(-3) = -4 + 9 = \boxed{5} \checkmark$$

$$D_y = \begin{vmatrix} 4 & 14 \\ 3 & 3 \end{vmatrix} = 4(3) - 3(14) = 12 - 42 = \boxed{-30} \checkmark$$

$$y = \frac{D_y}{D} = \frac{-30}{5} \quad \boxed{y = -6} \checkmark$$

Simplify

1) $\sqrt{64} = 8$

$8^2 = 64$ ✓

2) $-\sqrt{49} = -7$

$7^2 = 49$

3) $\sqrt{\frac{16}{25}} = \frac{4}{5}$

$\left(\frac{4}{5}\right)^2 = \frac{16}{25}$ ✓

4) $\sqrt{16+9} = \sqrt{25} = \boxed{5}$

5) $\sqrt{16} + \sqrt{9} = 4 + 3 = \boxed{7}$

Square-root Function

$f(x) = \sqrt{x}$

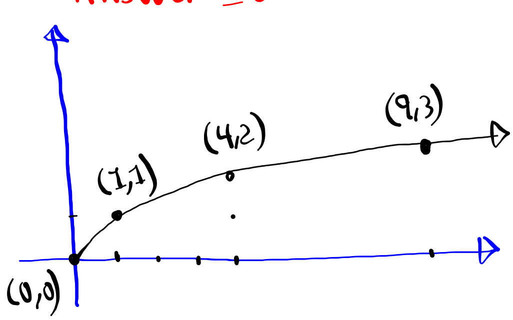
No index → index = 2

even index

Radicand ≥ 0 → $x \geq 0$

Answer ≥ 0

x	y
0	0
1	1
4	2
9	3
16	4



Domain $[0, \infty)$

Range $[0, \infty)$

Function by V.L.T.

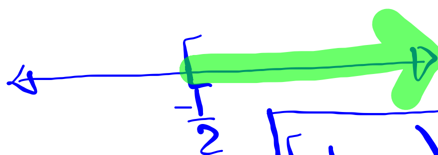
$f(x) = \sqrt{2x+1}$ Find

$f(0) = \sqrt{2(0)+1}$
$= \sqrt{0+1}$
$= \sqrt{1} = \boxed{1}$

$f\left(-\frac{1}{2}\right) = \sqrt{2\left(-\frac{1}{2}\right)+1}$
$= \sqrt{-1+1} = \sqrt{0} = \boxed{0}$

$f(4) = \sqrt{2(4)+1}$
$= \sqrt{8+1}$
$= \sqrt{9} = \boxed{3}$

Domain
Radicand ≥ 0
$2x+1 \geq 0$
$2x \geq -1$
$x \geq -\frac{1}{2}$



$\boxed{\left[-\frac{1}{2}, \infty\right)}$

Find the domain:

even index
Radicand ≥ 0

1) $f(x) = \sqrt{4-x}$
even index
$4 - x \geq 0$
$-x \geq -4$
$\boxed{x \leq 4}$
$(-\infty, 4]$

Odd index
No restrictions
All Reals
$(-\infty, \infty)$

2) $f(x) = \sqrt[4]{x-3}$
even index
$x - 3 \geq 0$
$x \geq 3$ $\boxed{[3, \infty)}$

3) $f(x) = \sqrt[3]{2x-5}$
odd index $\rightarrow$ All Reals $\boxed{(-\infty, \infty)}$

$\sqrt[n]{x^n} = x$

Assume $x \geq 0$

$y \geq 0$

$\sqrt[n]{xy} = \sqrt[n]{x}\,\sqrt[n]{y}$

$\sqrt[n]{\frac{x}{y}} = \frac{\sqrt[n]{x}}{\sqrt[n]{y}}$ $y>0$

$\sqrt[3]{\frac{x^3}{27}} = \frac{\sqrt[3]{x^3}}{\sqrt[3]{27}} = \frac{\cancel{\sqrt[3]{x^3}}}{\cancel{\sqrt[3]{3^3}}} = \boxed{\frac{x}{3}}$

Simplify

$\sqrt[4]{x^4} = \boxed{x}$

$\sqrt[5]{x^5} = \boxed{x}$

$\sqrt[2]{x^2} = \boxed{x}$

$\sqrt{16x^2} = \boxed{4x}$

$(4x)^2 = 16x^2$

Radical notation & Rational Exponent

$\sqrt[n]{x^m} = x^{\frac{m}{n}}$ $x \geq 0$

$\sqrt[5]{x^3} = x^{\frac{3}{5}}$

$\sqrt[7]{x^4} = x^{4/7}$

$x^{\frac{1}{2}} = \sqrt[2]{x^1} = \sqrt[2]{x}$

$= \sqrt{x}$

$x^{\frac{1}{3}} = \sqrt[3]{x^1} = \sqrt[3]{x}$

$20^{\frac{3}{2}} = \sqrt[2]{20^3} = \sqrt{20^3} = \sqrt{8000}$

$= \sqrt{100 \cdot 80}$

$= \sqrt{100} \cdot \sqrt{80}$

$= 10\,\sqrt{16 \cdot 5}$

$= 10 \cdot \sqrt{16}\,\sqrt{5}$

$= 10 \cdot 4\,\sqrt{5} = \boxed{40\sqrt{5}}$

Simplify

$$4^{\frac{5}{2}} - 8^{\frac{2}{3}} = \sqrt{4^5} - \sqrt[3]{8^2}$$

$$2^3 = 8$$

From Algebra

$$(x^m)^n = x^{m \cdot n}$$

$$= [2^2]^{5/2} - [2^3]^{2/3}$$

$$= 2^{\cancel{2} \cdot \frac{5}{\cancel{2}}} - 2^{\cancel{3} \cdot \frac{2}{\cancel{3}}}$$

$$= 2^5 - 2^2 = 32 - 4 = \boxed{28}$$

$$= \sqrt{4 \cdot 4 \cdot 4 \cdot 4 \cdot 4} - \sqrt[3]{8 \cdot 8}$$

$$= \sqrt{4}\sqrt{4}\sqrt{4}\sqrt{4}\sqrt{4} - \sqrt[3]{8}\sqrt[3]{8}$$

$$= 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 - 2 \cdot 2$$

$$= 32 - 4 = \boxed{28}$$

Recall $x^m \cdot x^n = x^{m+n}$

$$\frac{x^m}{x^n} = x^{m-n}$$

$$(x^m)^n = x^{m \cdot n}$$

Simplify $\sqrt[3]{x^2} \cdot \sqrt[5]{x} = x^{\frac{2}{3}} \cdot x^{\frac{1}{5}}$

$$= x^{\frac{2}{3} + \frac{1}{5}} = x^{\frac{13}{15}}$$

$$= \boxed{\sqrt[15]{x^{13}}}$$

Simplify $\dfrac{\sqrt[4]{x^3}}{\sqrt[5]{x^2}} = \dfrac{x^{\frac{3}{4}}}{x^{\frac{2}{5}}} = x^{\frac{3}{4}-\frac{2}{5}}$

$= x^{\frac{7}{20}} = \boxed{\sqrt[20]{x^7}}$

Simplify $\sqrt[5]{\sqrt[4]{x^3}}$

$= \sqrt[5]{x^{\frac{3}{4}}} = \left(x^{\frac{3}{4}}\right)^{\frac{1}{5}} = x^{\frac{3}{4}\cdot\frac{1}{5}}$

$\sqrt[20]{x^3} \longleftarrow = x^{\frac{3}{20}}$

Simplify

$\sqrt{AB} = \sqrt{A}\sqrt{B}$

$\sqrt{A}\sqrt{B} = \sqrt{AB}$

$A \geq 0,\ B \geq 0$

1) $\sqrt{50} = \sqrt{25\cdot 2}$
$= \sqrt{25}\sqrt{2} = \boxed{5\sqrt{2}}$

② $\sqrt{3}\sqrt{6} = \sqrt{3\cdot 6} = \sqrt{18}$
$= \sqrt{9\cdot 2} = \sqrt{9}\sqrt{2} = \boxed{3\sqrt{2}}$

③ $3\sqrt{15}\cdot 5\sqrt{6}$

$= 3\cdot 5\cdot \sqrt{15\cdot 6} = 15\sqrt{90}$

$= 15\sqrt{9\cdot 10} = 15\sqrt{9}\sqrt{10}$

$= 15\cdot 3\cdot \sqrt{10} = \boxed{45\sqrt{10}}$

Distribute and Simplify

$\sqrt{7}(\sqrt{14} - \sqrt{7})$

$= \sqrt{7}\sqrt{14} - \sqrt{7}\sqrt{7}$

$= \sqrt{98} - \sqrt{49}$

$= \sqrt{49 \cdot 2} - \sqrt{49} = \sqrt{49}\sqrt{2} - \sqrt{49} = \boxed{7\sqrt{2} - 7}$

FOIL and Simplify

$(5\sqrt{2} + 2\sqrt{3})(4\sqrt{2} - 3\sqrt{3})$

$= 20\sqrt{4} - 15\sqrt{6} + 8\sqrt{6} - 6\sqrt{9}$

$= 20 \cdot 2 \quad -7\sqrt{6} \quad -6 \cdot 3$

$= 40 - 7\sqrt{6} - 18 = \boxed{22 - 7\sqrt{6}}$

$(\sqrt{x})^2 = x$

$(\sqrt[3]{x})^3 = x \quad \Rightarrow (\sqrt[n]{x})^n = x$

$(\sqrt[4]{x})^4 = x$

Assume $x \geq 0$

Solve $\sqrt{3x-2} = 4$

$(\sqrt{3x-2})^2 = (4)^2$

$3x - 2 = 16$

$3x = 18$

$x = 6$

Check $\sqrt{3(6)-2} = 4 \rightarrow \sqrt{18-2} = 4$

$\sqrt{16} = 4$

$4 = 4$ ✓

Isolated Radical

index = 2

Square both Sides

Always Check Your Ans.

$\{6\}$

Solve $\sqrt{5x-1} - 8 = 0$

$$\sqrt{5x-1} = 8$$

$$\left(\sqrt{5x-1}\right)^2 = (8)^2$$

$$5x-1 = 64$$

$$5x = 65$$

$$\boxed{x = 13}\checkmark$$

$$\{13\}$$

Solve

$$\sqrt{2x+5} + 11 = 6$$

$$\sqrt{2x+5} = 6 - 11$$

$$\sqrt{2x+5} = -5$$

even root $\neq$ − #

$$\left(\sqrt{2x+5}\right)^2 = (-5)^2$$

$$2x+5 = 25$$

$$2x = 20$$

$$x = 10$$

it does not work

$$\sqrt{2(10)+5} + 11 = 6$$

$$\sqrt{20+5} + 11 = 6$$

$$\sqrt{25} + 11 = 6$$

$$5 + 11 = 6$$

$$16 = 6$$

False

$$\varnothing$$

$x=10$ is an extraneous solution.

Solve

$$\sqrt[3]{6x-3} - 3 = 0$$

$$\sqrt[3]{6x-3} = 3$$

index = 3

cube both Sides

$$\left(\sqrt[3]{6x-3}\right)^3 = (3)^3$$

$$6x - 3 = 27$$

$$6x = 30$$

$$\boxed{x = 5} \checkmark$$

$$\{5\}$$

Solve

$$x = \sqrt{6x+7}$$

Isolated radical

Index = 2

Square both Sides

$$(x)^2 = \left(\sqrt{6x+7}\right)^2$$

$$x^2 = 6x + 7$$

$$x^2 - 6x - 7 = 0$$

$$(x-7)(x+1) = 0$$

$$x - 7 = 0 \qquad x + 1 = 0$$

$$\boxed{x = 7} \checkmark \qquad \boxed{x = -1}$$

Does not work

$$\{7\}$$

Extraneous Solution -1

Class QZ 27

Show work
Box Your
Final Answer

$f(x) = \sqrt{6-2x}$

Portrait
style
only

Find

$f(-15) = \sqrt{6-2(-15)} = \sqrt{6+30} = \sqrt{36} = \boxed{6}$

$f(5) = \sqrt{6-2(5)} = \sqrt{6-10} = \sqrt{-4}$ $\boxed{\text{undefined}}$